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Pion Form Factor in the NLC QCD SR approach *

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Abstract

We present results of a calculation of the electromagnetic pion form factor within a framework of QCD Sum Rules with nonlocal condensates and using a perturbative spectral density which includes $\mathcal{O}(\alpha_s)$ contributions.

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I. INTRODUCTION

An archetypical example of a QCD (hadronic) observable is the pion form factor, which is typical for a hard-scattering process obeying a factorization theorem [1, 2]. Consequently, at asymptotically large Q^2 it can be cast in terms of a scale-dependent pion distribution amplitude (DA) [3] of leading twist two $\varphi_\pi(x, Q^2)$ convoluted with the hard-scattering amplitude of the process which contains the large external scale Q^2:

$$F_\pi^{\rm pert}(Q^2) = \frac{8\pi\alpha_s(Q^2) f_\pi^2}{9\,Q^2}\left|I_{-1}^\pi(Q^2)\right|^2 \quad \text{with} \quad I_{-1}^\pi(Q^2) = \int\limits_0^1 \frac{\varphi_\pi(x,Q^2)}{x}\,dx\,. \tag{1}$$

The nonperturbative input—the pion DA $\varphi_\pi(x, \mu^2)$—can be expressed as an expansion over Gegenbauer polynomials

$$\varphi_\pi(x,\mu^2) = \varphi^{\rm as}(x)\left[1 + \sum_{n\geq 1} a_{2n}(\mu^2)\, C_{2n}^{3/2}(2x-1)\right], \quad I_{-1}^\pi(\mu^2) = 3\left[1 + \sum_{n\geq 1} a_{2n}(\mu^2)\right], \tag{2}$$

where the asymptotic pion DA has the form

$$\varphi^{\rm as}(x) = 6\,x\,(1-x)\,, \tag{3}$$

while the scale dependence of coefficients $a_{2n}(\mu^2)$ is controlled by the ERBL evolution equation [1, 2].

At the one-loop level and at asymptotically large Q^2, the pion form factor simplifies to $F_\pi^{\rm pert}(Q^2) = 8\,\pi\,\alpha_s(Q^2)\,f_\pi^2/Q^2$ The onset of the asymptotic regime cannot be determined precisely; estimates [4, 5] show that this transition scale is of the order of 100 GeV^2.

On the other hand, at intermediate momentum transfers 20 GeV$^2 \geq Q^2 \geq 1$ GeV^2, the situation is more complicated because of the interplay of perturbative and nonperturbative effects. The latter effects are contained in a non-factorizable part—called the soft contribution—so that one has to take it into account using some nonperturbative concepts, e.g., the method of QCD sum rules (SR) [6, 7, 8], the local quark-hadron duality (LD) approach [6, 9], and others. Note in this context that, describing the pion form factor within the three-point QCD SR approach [6, 7], the shape of the pion DA becomes irrelevant. This considerably reduces the inherent theoretical uncertainty of the method. The same applies to the LD approach, but the latter contains an additional uncertainty related to the $s_0(Q^2)$ setting for intermediate and large values of Q^2—see for a discussion in [5].

However, the standard QCD SR [6, 7] are plagued by instabilities arising at $Q^2 \gtrsim 3$ GeV^2, which are induced by those terms in the operator product expansion that are either constant or grow linearly with Q^2 (see Tab. I). Such terms do not represent a nonperturbative contribution correctly. The corresponding diagrams result from the substitution of some propagators by constant factors that denote condensates lacking a Q^2-dependence, viz., $\langle T(q(z)\bar{q}(0))\rangle \to \langle \bar{q}(0)q(0)\rangle$. The scale dependence is retrieved by including in the calculation the contributions stemming from higher-dimension operators, like $\langle \bar{q}(0)D^2q(0)\rangle$, $\langle \bar{q}(0)(D^2)^2q(0)\rangle$ *etc.*, that are entailed by the Taylor expansion of the original nonlocal condensate (NLC), $\langle \bar{q}(0)q(z)\rangle$, being the nonperturbative part of the quark propagator. In order to obtain the correct large-Q^2 behavior and ensure that the total condensate contribution decreases for large Q^2, one has to resum all terms of the standard OPE bearing terms of the

sort $(Q^2/M^2)^n$. This is a rather tedious task and, therefore, we refrain from using the original Taylor expansion in our analysis and take instead recourse to a modified diagrammatic technique which makes use of new lines and vertices associated with NLC (details can be found in [8]).

TABLE I: Q^2-behavior of the nonperturbative contribution in different QCD SR approaches. Here, $c_1,\ c_2,\ c_3,\ c_4$ are dimensionless constants (not depending on Q^2). The abbreviations used are: LD for local duality, LO for leading order, and NLO for next-to-leading order, while λ_q^2 and M^2 denote the vacuum quark virtuality and the Borel parameter, respectively.

Approach	Accuracy	Condensates	Q^2-behavior of $\Phi_{\rm OPE}$
Standard QCD SR [6, 7]	LO	Local	$c_1 + Q^2/M^2$
QCD SR with NLCs [8]	LO	Local + Nonlocal	$\left(c_2 + Q^2/M^2\right)\left(e^{-c_3 Q^2 \lambda_q^2/M^4} + c_4\right)$
LD SR($M^2 \to \infty$) [5]	NLO	—	0
This paper	NLO	Nonlocal	$\left(c_1 + Q^2/M^2\right)\, e^{-c_3 Q^2 \lambda_q^2/M^4}$

An earlier attempt to generalize the QCD SR [8] approach by employing such NLC contributions turned out to be incomplete, because it was found to contain contributions originating from local condensates. This is related to the fact that a specific model (15) for the 3-point quark-gluon-antiquark NLC was used in which the NLC $M_i(x^2, y^2, (x-y)^2)$ are nonlocal only with respect to one single separation, say, x^2, out of the three possible inter-parton separations x^2, y^2, and $(x-y)^2$. As a result, also this type of approach suffers from the same shortcomings as the standard QCD SR. In contrast, LD SR have no condensate contribution due to the $M^2 \to \infty$ limit. The only trace of all contributing condensate contributions is embodied in the parameter s_0, which can be derived from the LD sum rule for f_π. Due to the Ward identity, these sum rules are connected only at $Q^2 = 0$, so that the applicability of this method to the pion form factor is actually confined to low momenta around $Q^2 \ll s_0$. The definition of s_0 at large Q^2 is not settled in this approach [5].

In this presentation, we report upon an investigation of the electromagnetic pion form factor which employs QCD SR with NLC [8, 9]. This enables us to enlarge the region of applicability of the QCD SR to momenta as high as 10 GeV^2. Moreover, we use a spectral density which includes terms of $\mathcal{O}(\alpha_s)$. The influence of this NLO contribution to the pion form factor reaches the level of 20%. The remainder of this report is organized as follows. The next section contains the necessary ingredients of the QCD SR approach with NLC. The second part of this section contains also our results. Our conclusions are given in section III, where we further discuss our findings in comparison with the available experimental data, lattice simulations, and other theoretical calculations.

II. PION FORM FACTOR FROM QCD SUM RULES WITH NONLOCAL CONDENSATES

The nonlocality of the QCD vacuum, suggested in [10, 11, 12, 13], is crucial for a correct determination of the pion DA and the computation of the pion form factor [8, 9]. For that

reason, let us first recall the main elements of this approach and discuss its application to the calculation of 3-point correlators in QCD.

For the scalar and vector condensates, we employ the same Gaussian model as in [12, 13], i.e.,

$$\langle\bar{q}(0)q(z)\rangle = \langle\bar{q}q\rangle\, e^{-|z^2|\lambda_q^2/8}\,;\ \langle\bar{q}(0)\gamma_\mu q(z)\rangle = \frac{i\, z_\mu\, z^2}{4}\, A_0\; e^{-|z^2|\lambda_q^2/8}\,, \tag{4}$$

where $A_0 = 2\alpha_s\pi\langle\bar{q}q\rangle^2/81$. Note that above, a Fock–Schwinger string is attached in-between the quark-antiquark fields in order to preserve gauge invariance. But adopting the fixed-point (Fock–Schwinger) gauge $z^\mu A_\mu(z) = 0$ each string reduces to unity, provided the integration path in the exponent is a straight line going from 0 to z. The nonlocality parameter $\lambda_q^2 = \langle k^2\rangle$ provides a useful measure of the average momentum of quarks in the QCD vacuum. It has been estimated in QCD SR [14, 15] and on the lattice [16, 17] with a value around $\lambda_q^2 = 0.45\pm0.1$ GeV^2. To parameterize the vector (V) and the axial-vector (A) quark-gluon-antiquark condensate, we use the expressions derived in [10]:

$$\begin{aligned}
\langle\bar{q}(0)\gamma_\mu(-g\widehat{A}_\nu(y))q(x)\rangle &= (y_\mu x_\nu - g_{\mu\nu}(y\cdot x))\overline{M}_1(x^2,y^2,(y-x)^2)\\
&+ (y_\mu y_\nu - g_{\mu\nu}y^2)\overline{M}_2(x^2,y^2,(y-x)^2)\,,\\
\langle\bar{q}(0)\gamma_5\gamma_\mu(-g\widehat{A}_\nu(y))q(x)\rangle &= i\varepsilon_{\mu\nu yx}\overline{M}_3(x^2,y^2,(y-x)^2)
\end{aligned}$$

with

$$\overline{M}_i(x^2,y^2,z^2) = A_i \iiint\limits_0^\infty d\alpha\, d\beta\, d\gamma\, f_i(\alpha,\beta,\gamma)\, e^{\left(\alpha x^2+\beta y^2+\gamma z^2\right)/4}\,, \tag{5}$$

where the following abbreviation $A_{1,2,3} \equiv A_0\times\left(-\frac{3}{2},2,\frac{3}{2}\right)$ was used. The minimal Gaussian model of the nonlocal QCD vacuum is introduced by the following ansatz

$$f_i^{\rm min}(\alpha,\beta,\gamma) = \delta(\alpha-\Lambda)\;\delta(\beta-\Lambda)\;\delta(\gamma-\Lambda) \tag{6}$$

with $\Lambda = \lambda_q^2/2$. This model violates the QCD equations of motion, while at the same time the 2-point correlator of the vector currents is not transverse. To restore the QCD equations of motion and to minimize the non-transversity of the VV correlator, an improved model of the QCD vacuum was proposed [18]:

$$f_i^{\rm imp}(\alpha,\beta,\gamma) = (1+X_i\partial_x+Y_i\partial_y+Y_i\partial_z)\,\delta(\alpha-x\Lambda)\,\delta(\beta-y\Lambda)\,\delta(\gamma-z\Lambda)\ . \tag{7}$$

Here $\Lambda = \frac{1}{2}\lambda_q^2$ and $z = y$, whereas

$$X_1 = +0.082\,;\ X_2 = -1.298\,;\ X_3 = +1.775\,;\ x = 0.788\,, \tag{8a}$$
$$Y_1 = -2.243\,;\ Y_2 = -0.239\,;\ Y_3 = -3.166\,;\ y = 0.212\,. \tag{8b}$$

These parameters satisfy the supplementary conditions

$$12\,(X_2+Y_2) - 9\,(X_1+Y_1) = 1\,, \qquad x+y = 1\,, \tag{9}$$

following from the QCD equations of motion.

The Borel SR for the pion form factor, based on the 3-point AAV correlator, was considered for local condensates in [6, 7], whereas the NLC case was treated in [8]. This way, one obtains the following SR

$$f_\pi^2\, F_\pi(Q^2) = \int_0^{s_0}\!\!\int_0^{s_0} ds_1\, ds_2\ \rho_3(s_1, s_2, Q^2)\, e^{-(s_1+s_2)/M^2} + \Phi_G(Q^2, M^2) + \Phi_{\langle\bar{q}q\rangle}(Q^2, M^2)\,. \quad (10)$$

Note that as long as the condensate terms Φ_G and $\Phi_{\langle\bar{q}q\rangle}$ are not specified, this SR may have a local or nonlocal content. On the other hand, the perturbative 3-point spectral density is given by

$$\rho_3^{(1)}(s_1, s_2, Q^2) \ = \ \left[\rho_3^{(0)}(s_1, s_2, Q^2) + \frac{\alpha_s(Q^2)}{4\pi}\,\Delta\rho_3^{(1)}(s_1, s_2, Q^2)\right]. \quad (11)$$

The leading-order spectral density $\rho_3^{(0)}(s_1, s_2, t)$ is known since the early eighties [6, 7]. As regards the next-to-leading order (NLO) spectral density $\Delta\rho_3^{(1)}(s_1, s_2, Q^2)$, it has been obtained quite recently [19]. The phenomenological side of the SR contains the contribution which stems from higher resonances, modeled via the spectral density

$$\rho_{\rm HR}(s_1, s_2) = [1 - \theta(s_1 < s_0)\theta(s_2 < s_0)]\ \rho_3(s_1, s_2, Q^2) \quad (12)$$

and using the continuum-threshold parameter s_0. In order to improve the low-scale behavior of the pion form factor, we adopt a scheme, developed in [4, 20], and employ an analytic running coupling [21]

$$\alpha_s(Q^2) \ = \ \frac{4\pi}{b_0}\left(\frac{1}{\ln(Q^2/\Lambda_{\rm QCD}^2)} - \frac{\Lambda_{\rm QCD}^2}{Q^2 - \Lambda_{\rm QCD}^2}\right), \quad (13)$$

with $b_0 = 9$ and $\Lambda_{\rm QCD} = 300$ MeV.

For our discussion to follow, we use for the nonperturbative terms Φ_G and $\Phi_{\langle\bar{q}q\rangle}$ in the local-condensate case the following expressions [6, 7]:

$$\Phi_G^{\rm loc}(M^2) = \frac{\langle\alpha_s GG\rangle}{12\,\pi\, M^2}\,, \qquad \Phi_{\langle\bar{q}q\rangle}^{\rm loc}(Q^2, M^2) = \frac{104\, A_0}{M^4}\left(1 + \frac{2\,Q^2}{13\,M^2}\right). \quad (14)$$

These expressions, that are used in the standard QCD SR for the pion form factor, show a wrong behavior at large Q^2: (i) The quark contribution contains both a linearly growing term as well as a constant one. (ii) The gluon contribution is just a constant. On the other hand, the perturbative term on the right-hand side of Eq. (10) behaves at large Q^2 like s_0/Q^4 or M^2/Q^4. Hence, the SR becomes unstable for $Q^2 > 3$ GeV^2. But using the generalized QCD SR with NLC, [8], this deficiency should not appear. Alas, even this approach has a dark side, because it still uses in the analysis of the pion form factor a partially local parameterization of the quark-gluon-antiquark NLC. Indeed, the following parametric functions (5) have been used in [8] ($\Lambda = \lambda_q^2/2$):

$$f_i^{\rm BR}(\alpha, \beta, \gamma) \ = \ \delta\,(\alpha - x_{i1}\Lambda)\,\delta\,(\beta - x_{i2}\Lambda)\,\delta\,(\gamma - x_{i3}\Lambda)\ , \quad (15)$$

$$x_{ij} \ = \ \begin{pmatrix} 0.4 & 0 & 0.4 \\ 0 & 1 & 0.4 \\ 0 & 0.4 & 0.4 \end{pmatrix}.$$

The absence of nonlocality effects in (5) for the quark-antiquark separation y^2 ($i = 1$) and also for the (anti)quark-gluon separations x^2 and $(x - y)^2$ ($i = 2, 3$) is revealed by the zero elements in the matrix x_{ij}.

Note that the NLC contributions to the pion form factor, entering the SR (10), can still be used in connection with an improved version of the quark-gluon NLC because the expressions obtained in [8] have the form of a convolution in the α-representation of the NLC distribution functions $f_i(\alpha, \beta, \gamma)$ with model-independent coefficient functions. In the present work we apply the minimal (6) and the improved (7) Gaussian models of NLC. The contribution from the vector condensate to $\Phi_{\langle\bar{q}q\rangle}$ reads

$$\Delta\Phi_V(M^2, Q^2) = \frac{8\,A_0}{M^4}\left(2 + \frac{Q^2}{2\,M^2 - \lambda_q^2}\right)\exp\left[\frac{-\,Q^2\,\lambda_q^2}{2\,M^2\left(2\,M^2 - \lambda_q^2\right)}\right]. \tag{16}$$

This term indeed vanishes for large Q^2 and is controlled by the nonlocality parameter λ_q^2. The larger λ_q^2, the faster this contribution decreases with Q^2. The explicit expressions for the other condensate contributions are omitted here, but their schematic Q^2 dependence can be read off from Table I.

The pion form factor $F_\pi(M^2, s_0)$, as a function of two additional parameters M^2 (Borel parameter) and s_0 (continuum threshold), is given at each fixed value of Q^2 by SR Eq. (10). The parameter s_0 marks the boundary between the pion state and higher resonances (A_1, π', etc.). We select its value at each momentum transfer Q^2 by applying to the function $F_\pi(M^2, s_0)$ the minimal-sensitivity condition with respect to the auxiliary parameter M^2 in the fiducial interval of the SR. These intervals and the value of the pion decay constant f_π for each considered NLC model, notably the minimal and the improved Gaussian one, are taken from the corresponding 2-point NLC QCD SR: $f_\pi = 0.137$ GeV^2, $M_-^2 = 1$ GeV^2, and $M_+^2 = 1.7$ GeV^2 for the minimal Gaussian model [13], whereas for the improved one [18] one has $f_\pi = 0.142$ GeV^2, $M_-^2 = 1$ GeV^2, and $M_+^2 = 1.9$ GeV^2. The continuum threshold $s_0^{\rm SR}(Q^2)$, which minimizes the dependence of the right-hand side of (10), is fixed by the root-mean-square deviation $\chi^2(Q^2, s_0)$, Eq. (A.1), in the Borel-parameter interval $M^2 \in [M_-^2, M_+^2]$ at each value of Q^2. The results of this procedure are shown in the left panel of Fig. 1. Both models generate approximately constant values of $s_0(Q^2)$ in the whole region $Q^2 \in [1; 10]$ GeV^2 with slightly higher values in the case of the improved Gaussian model.

The SR result for the pion form factor is defined numerically as the average value of the right-hand side of SR (10) with respect to the Borel parameter $M^2 \in [M_-^2, M_+^2]$:

$$F_\pi^{\rm SR}(Q^2) \;=\; \frac{1}{M_+^2 - M_-^2}\int_{M_-^2}^{M_+^2} F(Q^2, M^2, s_0(Q^2))\,dM^2\,. \tag{17}$$

The obtained predictions for the pion form factor for both Gaussian NLC models with $\lambda_q^2 = 0.4$ GeV^2, are shown in the right panel of Fig. 1 as dashed and solid curves, respectively, in comparison with the lattice result of [22] (dark grey strip limited from above at approximately 5 GeV^2). These theoretical results are compared with the available experimental data [23, 24] and previous theoretical estimates [5, 6, 7]. We also show in this figure in the form of light grey strips the minimal theoretical uncertainties of the QCD SR results. The central curves of our predictions can be represented by the corresponding interpolation formulas:

$$F_{\pi;\rm Min}^{\rm SR}(Q^2 = x\ {\rm GeV}^2) \;=\; 1.64\,e^{-1.73\,x^{0.32}}\,x\,, \tag{18a}$$

$$F_{\pi;\rm Imp}^{\rm SR}(Q^2 = x\ {\rm GeV}^2) \;=\; e^{-0.528\,x^{0.8}}\,x\left(0.016\,x^2 - 0.065\,x + 0.58\right)\,, \tag{18b}$$

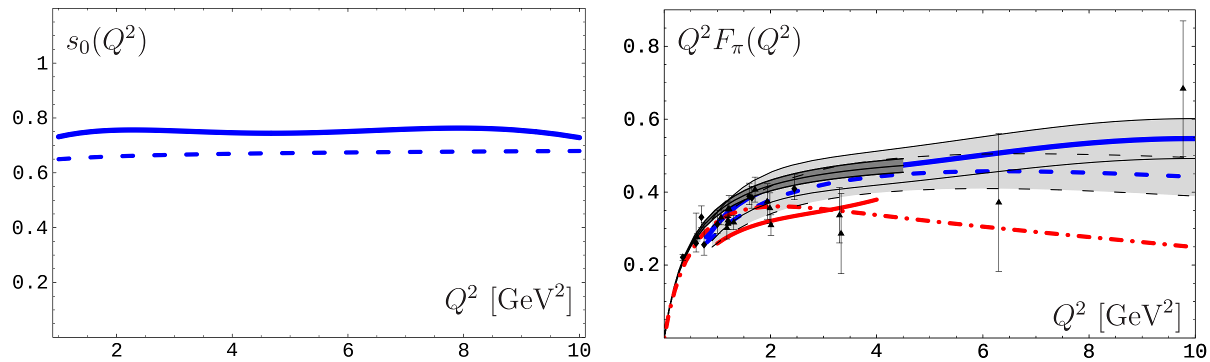

FIG. 1: Left panel: Continuum threshold $s_0(Q^2)$ [GeV^2] for the minimal (dashed line) and for the improved (solid line) NLC model. Right panel: Theoretical predictions for scaled pion form factor $Q^2F_\pi(Q^2)$ obtained by different methods and models. Dashed line—minimal NLC model; solid line—improved NLC model (in both cases $\lambda_q^2 = 0.4$ GeV^2 has been used and the corresponding uncertainties are indicated by light gray strips delimited by similar lines). The following designations are used: thick line between 1 and 4 GeV^2—standard QCD SR with local condensates [6, 7]; dash-dotted line—LD QCD SR [5]; triangles—Cornell experimental data [23]; diamonds—JLab experimental data [24]. The recent lattice result [22] is shown as a monopole fit containing error bars illustrated by a dark grey strip ending at ≈ 4.5 GeV^2.

valid for $Q^2 \in [1, 10]$ GeV^2, i. e., for $x \in [1, 10]$.

III. DISCUSSION AND CONCLUSIONS

We calculated the electromagnetic pion FF using QCD SR with NLC [8, 10] with a QCD vacuum nonlocality parameter $\lambda_q^2 = 0.4$ GeV^2 and using a perturbative spectral density proposed in [19]. This λ_q^2 value receives support from a recent comprehensive analysis [25, 26] of the CLEO data on the pion-photon transition.[1] The use of NLC enables one to considerably enlarge the region of applicability of the QCD SR towards momenta as high as 10 GeV^2—in contrast to the standard QCD SR approach [6, 7], where the SR can be applied only in the $Q^2 \leq 3$ GeV^2 region.

The main conclusions of our investigation can be summarized as follows:

- We found that the $O(\alpha_s)$-contribution influences the pion form factor at the level of 20%. This estimate is a little bit smaller than ones, obtained in [5, 19].

- We found that the central-line prediction of the improved model NLC model is inside the error strip of the minimal model up to $Q^2 = 6$ GeV^2. Therefore, we may conclude that both models are equally good in this region. In view of the absence of more precise experimental data on the pion form factor at present, we cannot give any preference to one or the other of the two considered NLC models.

[1] Using somewhat higher values of this parameter, would entail a decrease of the pion form factor owing to a stronger influence of the nonlocality effects. The opposite effect appears for smaller values of this parameter and leads to an increase of the pion form factor.

- It appears that our predictions are systematically higher than those of the LD approach [5]. This can be easily understood in terms of the effective LD threshold $s_0^{\rm LD}(Q^2)$. As we have already said, its value in the LD approach is well established only in the small-Q^2 region. For higher values, it is not firmly fixed; for instance, in [5] it was suggested to use a logarithmically increasing threshold

$$s_0^{\rm LD}(Q^2) = \frac{4\pi^2 f_\pi^2}{1+\alpha_s(Q^2)/\pi}\,,$$

  which is ≈ 0.62 GeV^2 for $Q^2 \approx 3$ GeV^2. We estimated that in order to imitate the NLC QCD SR results in the LD approach, one needs to use $s_0^{\rm LD}(Q^2 = 10~\text{GeV}^2) = 0.87~\text{GeV}^2$. This means that the $s_0^{\rm LD}$ uncertainty in the region of $Q^2 = 10$ GeV^2 is of the order of 30% This is the origin of the discussed difference between the LD results and ours.

- The lattice QCD results of [22] are in excellent agreement with our predictions.

- Both, the minimal and the improved Gaussian model for the NLC give results which are in good agreement within errors with the currently available experimental data.

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APPENDIX A: QCD SR PARAMETERS

The parameters of the NLC are $\Lambda = \lambda_q^2/2 = 0.2$ GeV^2, $\langle\alpha_s GG\rangle/\pi = 0.012$ GeV^4, and $\alpha_s\,\langle\bar{q}q\rangle^2 = 1.83\cdot 10^{-4}$ GeV^6. The nonlocal gluon-condensate contribution $\Phi_G(M^2)$ produces a very complicated expression. In analogy to the quark case, we model it by an exponential factor [8, 11]: $\Phi_G(M^2) = \Phi_G^{\rm loc}(M^2)\, e^{-\lambda_g^2 Q^2/M^4}$ with $\lambda_g^2 = 0.4$ GeV^2.

In order to determine the best value of the threshold s_0, we define the χ^2 function for each value of Q^2 and s_0 as follows

$$\chi^2(Q^2,s_0) = \frac{\varepsilon^{-2}}{N_M}\left[\sum_{i=0}^{N_M} Q^4\, F(Q^2,M_i^2,s_0)^2 - \frac{\left(\sum\limits_{i=0}^{N_M} Q^2\, F(Q^2,M_i^2,s_0)\right)^2}{N_M+1}\right]\,, \tag{A.1}$$

where we used $M_i^2 = M_-^2 + i\,\Delta_M$, $\Delta_M = (M_+^2 - M_-^2)/N_M$, $N_M = 10$, and with ε denoting the desired accuracy for $\chi^2 \simeq 1$ (the actual value used in the computation is $\varepsilon = 0.07$.)

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